

Quasi-particle properties and Cooper pairing in trapped Fermi gases

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The possibility for the particles in a Fermi gas to emit and reabsorb density and spin fluctuations gives rise to an effective mass and to a lifetime of the quasi-particles, as well as to an effective pairing interaction which affect in an important way the BCS critical temperature. We calculate these effects for a spherically symmetric trapped Fermi gas of ~ 1000 particles. The calculation provides insight on the many-body physics of finite Fermi gases and is closely related to similar problems recently considered in the case of atomic nuclei and neutron stars.

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The experimental advances in cooling and manipulation of atomic Fermi gases are not only important in their own right, but also because of the possibility of providing a deeper understanding of the physics of finite many-body Fermi systems. A crucial feature of atomic Fermi gases is the possibility of exploiting Feshbach resonances. These resonances occur when, due to the action of an external magnetic field, a two-body bound state crosses threshold. The result is a change in the sign of the two-body scattering length a and allows to explore Fermi superfluidity in all regimes from the weak-coupling limit, for $a < 0$ and $k_F|a| \ll 1$ (with k_F being the Fermi wavenumber) to the strong-coupling one, for $a < 0$ and $k_F|a| \sim 1$, as well as Bose-Einstein condensation of two-fermion dimers (for $a > 0$) [1]. This crossover from fermionic to bosonic superfluidity has been the object of extensive work in the recent years and the first experimental data are being collected [2]. A fundamental theoretical problem is the evaluation of the critical temperature and the $T = 0$ equation of state in the whole crossover region. In this paper we concentrate on the calculation of the effective quasi-particle interaction, mass and lifetime in a trapped Fermi gas as the resonance is approached from the fermionic, $a < 0$, side. As one moves closer to resonance the strength of the effective interaction increases and the quasi-particle properties are strongly renormalized due to the possibility of emitting and reabsorbing density and spin fluctuations. This problem is particularly important when studying fermionic superfluidity, since both the superfluid critical temperature, T_c , and the amplitude of the $T = 0$ pairing gap depend strongly on the actual quasi-particle properties. These features are normally neglected in the standard crossover theories. As we shall show, they have a strong effect on T_c . In what follows we develop a framework to take into account the effective quasi-particle properties

and which is appropriate to finite size systems in a non-uniform isotropic trap. The case of uniform atomic Fermi gases has been dealt with in Ref. [3]. In what follows we show that the discrete level structure has a crucial influence on the problem, and point out the specific effects on T_c of the various quasi-particle properties. We relate our approach to other calculations carried out for different Fermi systems. In fact the role of polarization effects on the quasi-particle properties has been recently studied in a number of fields of physics as nuclear physics, neutron stars and ^3He (cf. e.g. [4, 5] and references therein).

We consider a Fermi gas of N particles in two internal states described by the Hamiltonian

$$\hat{H} = \sum_{\sigma=\uparrow,\downarrow} \int d^3r \hat{\psi}_{\sigma}^{\dagger}(\mathbf{r})(H_0 - \mu)\hat{\psi}_{\sigma}(\mathbf{r}) + g \int d^3r \hat{\psi}_{\uparrow}^{\dagger}(\mathbf{r})\hat{\psi}_{\downarrow}^{\dagger}(\mathbf{r})\hat{\psi}_{\downarrow}(\mathbf{r})\hat{\psi}_{\uparrow}(\mathbf{r}) \quad (1)$$

where $H_0 = -\hbar^2\nabla^2/2m_a + V_{ext}(r)$ with $V_{ext}(r) = m_a\omega_0 r^2/2$, ω_0 being the trap frequency, and m_a the atomic mass. We are here interested in the BCS side of the resonance where $g < 0$. We have furthermore assumed the number of particles and the external confining potentials V_{ext} to be equal for both internal states so that we also have $\mu_{\uparrow} = \mu_{\downarrow} = \mu$.

All the properties of the system are described in terms of the normal and anomalous propagators, which satisfy the Dyson equations (cf. e.g. ref. [5])

$$\mathcal{G}_{\uparrow\uparrow}(\omega_n) = \mathcal{G}_{\uparrow\uparrow}^0(\omega_n) + \mathcal{G}_{\uparrow\uparrow}^0(\omega_n)[\Sigma_{\uparrow\uparrow}(\omega_n)\mathcal{G}_{\uparrow\uparrow}(\omega_n) - W_{\uparrow\downarrow}(\omega_n)\mathcal{F}_{\uparrow\downarrow}^{\dagger}(\omega_n)], \quad (2)$$

$$\mathcal{F}_{\uparrow\downarrow}^{\dagger}(\omega_n) = \mathcal{G}_{\uparrow\downarrow}^0(-\omega_n)[W_{\uparrow\downarrow}^{\dagger}(\omega_n)\mathcal{G}_{\uparrow\uparrow}(\omega_n) + \Sigma_{\uparrow\downarrow}(-\omega_n)\mathcal{F}_{\uparrow\downarrow}^{\dagger}(\omega_n)], \quad (3)$$

where $\mathcal{G}_{\uparrow\uparrow}(\omega_n)$ stands for $\mathcal{G}_{\uparrow\uparrow}(\mathbf{x}, \mathbf{x}', \omega_n)$ and $\mathcal{G}_{\uparrow\uparrow}(-\omega_n)$ for $\mathcal{G}_{\uparrow\uparrow}(\mathbf{x}', \mathbf{x}, -\omega_n)$, and similarly for the other propagators. The quantity $\hbar\omega_n = (2n+1)\pi/\beta$ is a fermionic Matsubara frequency with $\beta^{-1} = k_B T$, T being the

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temperature of the system and k_B the Boltzmann constant. $\Sigma_{\sigma\sigma}$ and $W_{\sigma,-\sigma}$ are the normal and anomalous self-energies respectively, and the products of propagators and self-energies imply integration over the internal spatial coordinates. Completely analogous equations to (2) and (3) are satisfied by $\mathcal{G}_{\downarrow\downarrow}$ and $\mathcal{F}_{\downarrow\downarrow}$. The first step to be taken in carrying out the calculation, concerns the choice of a specific approximation for the self-energies. In this work we account for the possibility for particles to emit and reabsorb density and spin fluctuations and therefore assume $\Sigma_{\uparrow\uparrow}(\omega_n) = g n_{\downarrow}(\mathbf{x})\delta(\mathbf{x} - \mathbf{x}') + \Sigma_{\uparrow\uparrow}^{\text{ph}}(\omega_n)$. The first term corresponds to the usual ω -independent Hartree self-energy. Treating the phononic contribution $\Sigma_{\uparrow\uparrow}^{\text{ph}}$ to the self-energy in the Random Phase Approximation (RPA) one finds: $\Sigma_{\uparrow\uparrow}^{\text{ph}}(\omega_n) = -(g^2/4\beta) \sum_m \mathcal{G}^0(\omega_n + \omega_m) [\Pi_\rho(\omega_m) + 3\Pi_{\sigma_z}(\omega_m) - 2\Pi_0(\omega_m)]$. Here Π_0 is the density correlation function in the Hartree approximation, while Π_ρ and Π_{σ_z} are the density and spin correlation functions respectively in the RPA. The relation between these functions follows introducing the Hartree correlation function for a single species χ_0 , to find $\Pi_0 = 2\chi_0$ together with $\Pi_\rho = 2\chi_0/[1 - g\chi_0]$ and $\Pi_{\sigma_z} = 2\chi_0/[1 + g\chi_0]$. Due to the isotropy of the system the spin-spin correlation functions in the x , y and z direction are equal, a fact which is accounted for by multiplying Π_{σ_z} by a factor of 3. Finally we also used $\mathcal{G}_{\uparrow\uparrow}^0 = \mathcal{G}_{\downarrow\downarrow}^0 = \mathcal{G}^0$. The Hartree density correlation function admits the Lehmann representation

$$\Pi^0(\mathbf{x}, \mathbf{x}', \omega_m) = \int_{-\infty}^{+\infty} \frac{d\omega'}{2\pi} \frac{\pi^0(\mathbf{x}, \mathbf{x}', \omega')}{i\omega_m - \omega'}, \quad (4)$$

with

$$\pi^0(\mathbf{x}, \mathbf{x}', \omega') = \sum_{ij} \frac{e^{-\beta E_i} - e^{-\beta E_j}}{Z} \times \langle i | \hat{\rho}(\mathbf{x}) | j \rangle \langle j | \hat{\rho}(\mathbf{x}') | i \rangle \delta(\hbar\omega' - E_i + E_j). \quad (5)$$

In Eq. (5) $\hat{\rho}(\mathbf{x}) = \hat{\psi}_{\uparrow}^{\dagger}(\mathbf{x})\hat{\psi}_{\uparrow}(\mathbf{x}) + \hat{\psi}_{\downarrow}^{\dagger}(\mathbf{x})\hat{\psi}_{\downarrow}(\mathbf{x})$, while i and j are the many-body eigenstates in the Hartree approximation (particle-hole excitations), E_i and E_j are the corresponding energies and ω_m is a bosonic Matsubara frequency. Π_ρ admits a similar representation with i and j replaced by the many-body eigenstates in the RPA approximation (collective modes). The same is true for Π_{σ_z} where in addition $\hat{\rho}$ must be replaced by $\hat{\sigma}_z = \hat{\psi}_{\uparrow}^{\dagger}(\mathbf{x})\hat{\psi}_{\uparrow}(\mathbf{x}) - \hat{\psi}_{\downarrow}^{\dagger}(\mathbf{x})\hat{\psi}_{\downarrow}(\mathbf{x})$. The anomalous self-energy, on the other hand, is given by $W_{\uparrow\downarrow}(\omega_n) = -\beta^{-1} \sum_m V_{\uparrow\downarrow}^{\text{eff}}(\omega_m) \mathcal{F}_{\uparrow\downarrow}(\omega_n + \omega_m)$, where $V_{\uparrow\downarrow}^{\text{eff}}(\omega_m) = g\delta(\mathbf{x} - \mathbf{x}') + (g^2/4)[\Pi_\rho(\omega_m) - 3\Pi_{\sigma_z}(\omega_m)]$ is the effective quasi-particle interaction including the exchange of density and spin modes.

In a general spherically symmetric system it is convenient to express the Dyson equations in the Hartree-Fock (spherical) basis of eigenstates $\phi_\nu(\mathbf{x}) = R_{nl}^{\text{HF}}(r)Y_{lm}(\Omega)$. Here ν stands for n, l, m (radial, angular momentum

and magnetic quantum numbers) and Y_{lm} is a spherical harmonic. The corresponding energy measured from the Fermi level is denoted by $\xi_\nu = \xi_{nl}$ independent of m . We shall also omit the spin indices. Keeping the matrix elements of the effective interaction only between states connected by the time reversal $V_{\nu\nu'}^{\text{eff}}(\omega_m) \equiv \langle \nu\bar{\nu} | V^{\text{eff}}(\omega_m) | \nu'\bar{\nu}' \rangle$ and solving the Dyson equations in the proximity of T_c one obtains

$$W_\nu(\omega_n) = - \sum_{\nu'} \frac{1}{\beta} \sum_m V_{\nu\nu'}^{\text{eff}}(\omega_m) \mathcal{F}_{\nu'}(\omega_n + \omega_m), \quad (6)$$

with $\mathcal{F}_\nu(\omega_n) = W_\nu(\omega_n)G_\nu(\omega_n)G_\nu(-\omega_n)$ and $G_\nu^{-1}(\omega_n) = i\hbar\omega_n - \xi_\nu - \Sigma_\nu^{\text{ph}}(\omega_n)$.

Introducing the Lehmann representation for the Π 's yields the following representation for the effective interaction:

$$V_{\nu\nu'}^{\text{eff}}(\omega_m) = g_{\nu\nu'} - \int_0^\infty \frac{d\omega}{2\pi} v_{\nu\nu'}(\omega) \frac{2\omega}{\omega_m^2 + \omega^2} \quad (7)$$

where $g_{\nu\nu'} = \langle \nu\bar{\nu} | g\delta(\mathbf{x} - \mathbf{x}') | \nu'\bar{\nu}' \rangle$ is the matrix element of the bare interaction and $v_{\nu\nu'}(\omega) = (g^2/4) \langle \nu\bar{\nu} | [\pi_\rho(\mathbf{x}, \mathbf{x}', \omega) - 3\pi_{\sigma_z}(\mathbf{x}, \mathbf{x}', \omega)] | \nu'\bar{\nu}' \rangle$. Correspondingly, after summation over the internal bosonic Matsubara frequencies, one obtains for the self-energy

$$\Sigma_\nu^{\text{ph}}(\omega_n) = \sum_{\nu'} \int_0^\infty \frac{d\omega}{2\pi} \sigma_{\nu\nu'}(\omega) \times \left(\frac{1 + n_B(\hbar\omega) - n_F(\epsilon_{\nu'})}{i\omega_n - \hbar^{-1}\epsilon_{\nu'} - \omega} + \frac{n_B(\hbar\omega) + n_F(\epsilon_{\nu'})}{i\omega_n - \hbar^{-1}\epsilon_{\nu'} + \omega} \right) \quad (8)$$

with $\sigma_{\nu\nu'}(\omega) = (g^2/4) \langle \nu\bar{\nu} | [\pi_\rho(\mathbf{x}, \mathbf{x}', \omega) + 3\pi_{\sigma_z}(\mathbf{x}, \mathbf{x}', \omega) - 2\pi^0(\mathbf{x}, \mathbf{x}', \omega)] | \nu'\bar{\nu}' \rangle$, and $n_B(x) = (e^{\beta x} - 1)^{-1}$ and $n_F(x) = (e^{\beta x} + 1)^{-1}$ being the Bose and Fermi thermal distribution functions respectively [6]. Eqs. (6), (7) and (8) constitute a set of equations of the type found in the theory of strong-coupling superconductivity (see for instance [5, 7]) and should be solved self-consistently together with the number equation

$$N = \sum_{\sigma=\uparrow,\downarrow} \int d\mathbf{x} \lim_{\eta \rightarrow 0} \sum_n \mathcal{G}_{\sigma\sigma}(\mathbf{x}, \mathbf{x}, \omega_n) e^{i\omega_n \eta}, \quad (9)$$

which fixes the chemical potential. At low and intermediate densities one expects the single-pole approximation for the propagators to be accurate. Introducing this approximation for the spectral functions of G and \mathcal{F} (see for instance [8, 9]) one obtains

$$\mathcal{F}_\nu(\omega_n) \simeq - \frac{Z_\nu^2 W_\nu(\epsilon_\nu)}{2\epsilon_\nu} \times \left[\frac{1}{i|\hbar\omega_n| - \epsilon_\nu + i\gamma_\nu} - \frac{1}{i|\hbar\omega_n| + \epsilon_\nu + i\gamma_\nu} \right], \quad (10)$$

where $\epsilon_\nu = \xi_\nu + \text{Re}\Sigma_\nu^{\text{ph}}(\omega)|_{\hbar\omega=\epsilon_\nu}$ is the renormalized single-particle energy, $\gamma_\nu = \text{Im}\Sigma_\nu^{\text{ph}}(\omega)|_{\hbar\omega=\epsilon_\nu - i\eta}$ the level width, and $Z_\nu = (1 - \hbar^{-1}\partial\Sigma_\nu^{\text{ph}}(\omega)/\partial\omega|_{\hbar\omega=\epsilon_\nu})^{-1} \leq 1$ the

quasi-particle strength. The single-pole approximation is accurate as long as the resulting single-particle level width is small compared with the Hartree level spacing and Z_ν is not too small. At these densities one also expects the effect of the self-energies Σ^{ph} and W on the number equation to be small compared with the Hartree one. We thus approximated $\mathcal{G}_{\sigma\sigma}$ in Eq. (9) with the Hartree expression $\sum_\nu |\phi_\nu(\mathbf{x})|^2 / (i\hbar\omega_n - \xi_\nu)$. Calculations which go beyond the one pole approximation [10] and which include vertex corrections [11] have been carried out recently to determine the pairing gap of superfluid atomic nuclei. These calculations have shown that one recovers the results of the one-pole approximation for weak and intermediate coupling.

Several further approximation schemes have been used in the literature. Introducing Eq. (7) and (10) into Eq. (6) and neglecting the phonon contribution to the effective interaction one recovers the approximations of Ref. [12]. For a Fermi gas this is in general a very poor approximation since, for instance, in a uniform system the phononic contribution to the interaction produces a constant reduction factor $(4e)^{1/3} \simeq 2.2$ in the critical temperature in the weak-coupling limit [13]. On the other hand, if one keeps the phonon induced interaction and neglects the quasi-particle width one recovers the result of Ref. [9], found in the context of neutron matter. As we shall see, it is the quasi-particle width which has the largest effects on the critical temperature in the intermediate and strong coupling regimes. For these reasons we include both effects, together with the quasi-particle strength and the renormalization of the single particle energies, to get

$$\Delta_\nu = \sum_{\nu'} \left[\bar{g}_{\nu\nu'} - 2\hbar \int_0^\infty \frac{d\omega}{2\pi} \frac{Z_\nu v_{\nu\nu'}(\omega) Z_{\nu'}}{|\epsilon_\nu| + |\epsilon_{\nu'}| + \hbar\omega} \right] \frac{\Delta_{\nu'}}{2\epsilon_{\nu'}} \times h(\epsilon_{\nu'}, \gamma_{\nu'}, T) \quad (11)$$

with $\bar{g}_{\nu\nu'} = Z_\nu g_{\nu\nu'} Z_{\nu'}$ and $\Delta_\nu = Z_\nu W_\nu(\epsilon_\nu)$. The dimensionless function $h(\epsilon_{\nu'}, \gamma_{\nu'}, T)$ is given by [12]:

$$h(\epsilon_{\nu'}, \gamma_{\nu'}, T) = 2k_B T \sum_{n=0}^\infty \left[\frac{2\epsilon_{\nu'}}{(\hbar\omega_n + \gamma_{\nu'})^2 + \epsilon_{\nu'}^2} \right], \quad (12)$$

and reduces to the usual hyperbolic tangent of BCS theory when $\gamma_{\nu'} = 0$. The effective interaction is here treated in the Bloch-Horowitz approximation. This means that we have neglected the effect of the level width on the induced interaction and taken the absolute value of the external quasi-particle energy in the denominator. A detailed comparison between the results of this approximation and of the full self-consistent Gorkov theory was carried out in Ref. [10]. There it was shown that the two approaches lead to very similar results in the weak and intermediate coupling regimes. Consequently we kept this level of approximation in our calculations. Eq. (11) is an eigenvalue equation for Δ_ν and T_c is the highest temperature at which the eigenvalue 1 appears. Notice that the equation contains a sum over ν' which needs special care.

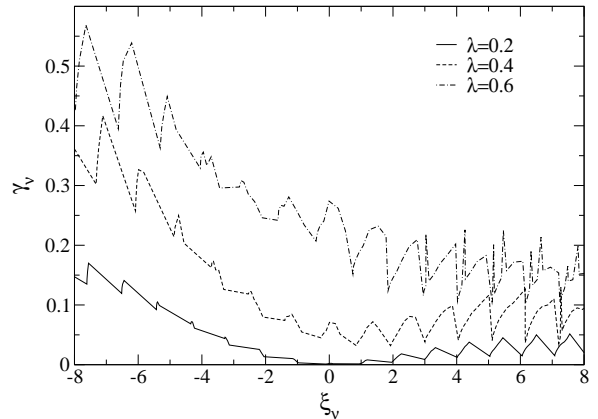


FIG. 1: γ_ν as a function of the Hartree quasi-particle energy ξ_ν for $\lambda = 0.2$ (solid line), $\lambda = 0.4$ (dashed line) and $\lambda = 0.6$ (dot-dashed line). The quantities are measured in units of $\hbar\omega_0$.

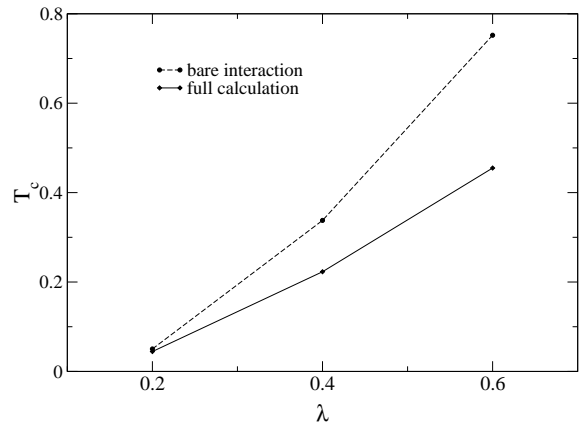


FIG. 2: T_c^{bare} and T_c (in units of $\hbar\omega_0$) as a function of the interaction strength λ . The values of T_c^{bare} are given in the caption of Table I, while those of T_c include all contributions in Eq. (11).

The contribution of the bare interaction, in fact, leads to an ultra-violet divergence, since it arises from a contact approximation to the true interatomic potential. Several approaches have been developed to eliminate this divergence, approaches which involve a renormalization of the coupling constant g or the introduction of a pseudopotential [14]. For a trapped gas, for which only the bare interaction is considered, the results of these calculations may be reproduced by introducing a cut-off in the sum at $\epsilon_{\nu'} = \epsilon_F$ [15]. Because the main scope of the present work is on the phonon induced effects, we have treated the bare interaction at this level of approximation. Due to the explicit dependence on $\epsilon_{\nu'}$, the phonon induced interaction has a natural cut-off. Numerical calculations have shown that the terms with $|\epsilon_{\nu'}| \gtrsim \epsilon_F$ give negligible

	$\lambda=0.2$	$\lambda=0.4$	$\lambda=0.6$
V^{eff}	0.92	0.86	0.90
$g + \text{Re}\Sigma^{\text{ph}}$	1.01	1.03	1.03
$g + Z$	0.98	0.90	0.93
$g + \gamma$	0.97	0.85	0.76
$V^{\text{eff}} + \text{Re}\Sigma^{\text{ph}} + \gamma + Z$	0.88	0.66	0.61

TABLE I: The table gives the ratio T_c/T_c^{bare} associated with the indicated contribution. T_c^{bare} is calculated including only g and we find the following results: for $\lambda = 0.2$, $k_B T_c^{\text{bare}} = 5.03 \times 10^{-2} \hbar \omega_0$, for $\lambda = 0.4$, $k_B T_c^{\text{bare}} = 0.34 \hbar \omega_0$ and for $\lambda = 0.6$, $k_B T_c^{\text{bare}} = 0.75 \hbar \omega_0$.

contributions.

Applying the formalism to a spherically symmetric infinite square well we found the results of Gorkov and Melik-Barkhudarov [13] in the weak-coupling limit. In the intermediate coupling regime we did not find the strong reduction predicted in Ref. [3]; this may be due to the different set of approximations used. For a harmonically confined gas the finite level spacing strongly affects the properties of the system, and we find the results reported in Table I. We have performed the calculations for a cloud of ~ 1000 particles and for three different coupling strengths λ . Here λ is defined in analogy with a uniform system as $|g|m_a k_F / 2\pi^2 \hbar^2$ with $k_F = (3\pi^3)^{1/3} [n(0)]^{1/3}$, and $n(0)$ being the central density of the cloud.

In the weak-coupling regime the effects of the induced interaction and self-energy are strongly suppressed by the discrete shell structure, and the reduction predicted by

Gorkov [13] for a uniform system is almost absent. This is because the largest contributions to the sum in Eq. (11) comes from the condition $\epsilon_\nu = \epsilon_{\nu'} = \omega = 0$, which corresponds to particles at the Fermi surface and to the exchange of a phonon with zero energy. The discrete level structure causes the values of the spectral functions relative to the Π functions (see Eq. (5)) to be negligible near $\omega = 0$, and consequently the effect of the phonon-induced interaction to be small.

The real part $\text{Re}\Sigma^{\text{ph}}$ causes an increase in the critical temperature since it leads to an effective increase of the density of levels at the Fermi energy, and thus to a stronger effective coupling strength. On the other hand, both Z_ν and γ_ν depress the value of T_c . The former causes a weaker effective interaction between quasi-particles, as can be deduced from Eq. (11) recalling that $Z_\nu \leq 1$ [16]. Because γ_ν measures the energy range over which the quasi-particle state is spread due to the coupling to vibrations (lifetime), its presence effectively inhibits pairing between quasi-particles [12]. In Fig. 1 we show the value of γ_ν for three different coupling strengths. Its effect is important in the intermediate coupling regime, as can be seen in Table I for the cases $\lambda = 0.4$ and $\lambda = 0.6$. In Fig. 2 are shown the values of T_c^{bare} , obtained when only the (bare) direct interaction g is included, and of T_c when all the terms in Eq. (11) are considered.

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